Supersonic Flutter of Laminated Circular Cylindrical Shell Panels

R. S. Srinivasan* and B. J. C. Babu†
Indian Institute of Technology, Madras 600 036, India

Introduction

COMPOSITE materials are used in the construction of curved panels of aircraft and space vehicles. Only a few works[1-3] are available on panel flutter analysis of shell panels.

In this investigation, circular cylindrical shell panels of rectangular planform, with edges clamped laterally and free from in-plane stresses (movable edges), are considered. The panel is subjected to supersonic flow parallel to the generator on one side only. The aerodynamic forces are approximated by using two-dimensional quasisteady aerodynamic theory. The problem is solved by using the integral equation technique.

Formulation

A typical cross-ply laminated panel is shown in the inset of Fig. 1. Two fourth-order aeroelastic governing differential equations, based on Donnell's shallow shell approximation, can be derived in terms of the transverse displacement w and stress function F.[4] They are

$$[A^*_{22}F'''' + (2A^*_{12}+A^*_{66})F''^{00}/R^2 + A^*_{11}F^{0000}/R^4] + [B^*_{21}w'''' + B^*_{12}w^{0000}/R^4 - w''/R] = 0 \tag{1}$$

$$[B^*_{21}F'''' + B^*_{12}F^{0000}/R^4 - F''/R] + [D^*_{11}w'''' + 2(D_{12}^* + 2D_{66}^*)w''^{00}/R^2 + D^*_{22}w^{0000}/R^4] = -\Delta p_z - \rho_m h\ddot{w} \tag{2}$$

where the superscripts $(\ )'$, $(\ )^0$, and $(\dot{\ })$ indicate differentiation with respect to x, θ, and time coordinates, respectively; $[A^*]=[A^{-1}]$; $[B^*]=[A^{-1}]\ [B]$; $[D^*]=([B][A^{-1}][B])-[D]$; $[A]$, $[B]$, $[D]$ correspond to the matrices of the constitutive equations of a layered panel; R is the radius of curvature of the panel; ρ_m the mass density of the panel material; and h the thickness of the panel. For Mach numbers $M_\infty > 1.6$, Δp_z is given by

$$\Delta p_z = -[2q/(M_\infty^2 - 1)^{1/2}] \times [w' + (1/V)(M_\infty^2 - 2) \times \dot{w}/(M_\infty^2 - 1)] \tag{3}$$

where q is the dynamic pressure and V the velocity of the airstream. The boundary conditions of the panel can be expressed as follows:

Along $x = 0$ and a,

$$w = w' = F = F' = 0$$

Along $\theta = 0$ and θ_0,

$$w = w^0 = F = F^0 = 0 \tag{4}$$

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*Professor, Department of Applied Mechanics.

†Senior Design Engineer, Fibre Reinforced Plastics Research Centre.

The shell panel is divided by lines parallel to the x and θ directions (see inset in Fig. 1). Let

$$w'''' = r(x,\theta), \qquad w^{0000} = s(x,\theta) \tag{5}$$

For points along the line $\theta = \theta_n$, Eq. (5) can be represented as

$$w(x,\theta_n) = \int_0^1 G(x,\xi,\theta_n)r(\xi,\theta_n)\,d\xi \tag{6}$$

where $G(x,\xi,\theta_n)$ is the Green's function of the boundary-value problem $w'''' = 0$, with the boundary conditions $w = w' = 0$ at $x = 0$ and a.

The integral equation (6) is approximated by sum as

$$w(x_m,\theta_n) = \sum_{i=1}^{\bar{M}} G(x_m,\xi_i,\theta_n)\alpha_i r_n(\xi_i,\theta_n) \qquad (m = 1,2,\ldots,\bar{M}) \tag{7}$$

where α_i are the coefficients of Simpson's quadrature rule, and $r_n(\xi_i, \theta_n)$ are the values of w'''' at grid points.

Assembling the equations for all the lines parallel to the θ axis yields $\{w\} = [H]\{r\}$, where $\{w\}$, $\{r\}$ are the values of w and r at all the grid points.

In a similar way, considering the lines parallel to the x axis, Eq. (5) can be written as $\{w\}^* = [J]\ \{s\}^*$, where the $(\ )^*$ indicates that the points are reckoned along the lines parallel to the x axis. The elements of $\{w\}^*$ and $\{s\}^*$ can be rearranged in the same order as $\{w\}$, which can be written as $\{w\} = [\bar{J}]\{s\}$. Thus, it can be shown that

$$\{r\} = [H]^{-1}[\bar{J}]\{s\} \tag{8}$$

Using the derivatives of Green's function and Eq. (8), the derivatives of $\{w\}$ can be obtained in terms of $\{s\}$.

In a similar manner, assuming $F'''' = k(x,\theta)$, $F^{0000} = \ell(x,\theta)$, and using Green's functions satisfying the boundary conditions pertaining to F, the derivatives of F can be expressed in terms of $\{\ell\}$ (for more details see Ref. 6).

Substituting these expressions for the derivatives in Eqs. (1) and (2), and assuming $\{\ell\} = \{\bar{\ell}\}e^{\omega t}$, $\{s\} = \{\bar{s}\}e^{\omega t}$, the governing equations can be written in the form

$$[K]_{11}\{\bar{\ell}\} + [K]_{12}\{\bar{s}\} = 0 \tag{9}$$

$$[K]_{12}\{\bar{\ell}\} + [K]_{22}\{\bar{s}\} + \beta[AK]\{\bar{s}\} = -(\Omega^2 + g_a\Omega)[M]\{\bar{s}\} \tag{10}$$

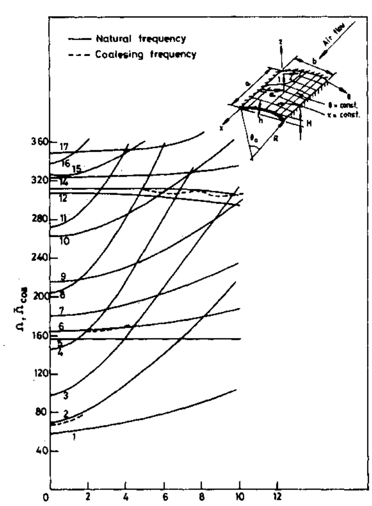

Fig. 1 Effect of rise on natural frequency for two-layered cross-ply panels, $a/b = 2$, $b/h = 100$, [0/90], $E_{11}/E_{22} = 25$, $G_{12}/E_{22} = 0.5$, $\nu_{12} = 0.25$.

where

$$\beta = \frac{2qa^3}{E_{22}h^3(M_\infty^2 - 1)^{1/2}}, \qquad \Omega^2 = \frac{\rho_m a^4}{E_{22}h^2}\omega^2$$

$$g_a = \frac{\beta h(M_\infty^2 - 2)}{aV(M_\infty^2 - 1)}\sqrt{\frac{E_{22}}{\rho_m}}$$

and where $[K]$ is the stiffness matrix, $[AK]$ the aerodynamic matrix, $[M]$ the mass matrix, and ω the complex quantity.

Eliminating $\{\bar{t}\}$ from Eqs. (9) and (10), one gets

$$([K]^* + \beta[AK])\{\bar{s}\} = \lambda[M]\{\bar{s}\} \tag{11}$$

where $[K]^* = ([K]_{22} - [K]_{12}[K]_{11}{}^{-1}[K]_{12})$ and $\lambda = -(\Omega^2 + g_a\Omega)$.

In general, λ can be complex. Hence, $\lambda = \lambda_R \pm i\lambda_I$. When flutter occurs, Ω is purely imaginary, i.e., $\Omega = i\bar{\Omega}$. Then

$$\lambda_R = \bar{\Omega}^2, \qquad \lambda_I = g_a\bar{\Omega} \tag{12}$$

In the absence of airflow (i.e., $\beta = 0$, $g_a = 0$), Eq. (11) represents a free vibration problem. Using two-dimensional quasistatic aerodynamic approximation (i.e., $g_a = 0$), when β is gradually increased, some of the eigenvalues approach each other and, for a certain value β_{coa}, two of them coalesce at $\bar{\Omega}$ (as an example). If damping is considered, the onset of dynamic instability occurs at a value of β ($= \beta_{cr}$) higher than β_{coa}.

Numerical Work

By increasing the mesh size, the convergence of flutter boundaries was studied and it was decided to adopt 11 × 11 mesh for further numerical work. In Figs. 1 and 2, the natural frequencies Ω and the first coalescence frequency $\bar{\Omega}_{coa}$ vs shell rise are presented. It may be noted that as the shell rise increases, the coalescence frequency $\bar{\Omega}_{coa}$ jumps from a lower value to the higher one in the regions $(H/h) = 2$–3 and 4–6. This trend is similar to the one observed by Matsuzaki[7] in the case of flutter of isotropic curved panels.

Since no reference is available to check the numerical results, Eqs. (1) and (2) were solved by using Galerkin's method. In this procedure, the expressions for w and F were assumed as double series consisting of characteristic functions corresponding to clamped-clamped beams, which satisfy the boundary conditions of the panel exactly. These expressions were substituted in Eqs. (1) and (2) and, after following Galerkin's procedure, lead to algebraic equations. Solving them as before, the flutter boundaries can be found out. For example, for the case of two-layered panels $a/b = 2$ and $H/h = 0$–10, the flutter bounds have been determined and marked in Fig. 3, where they agree with the flutter boundary values obtained using an integral equation technique, thus validating the results of this analysis.

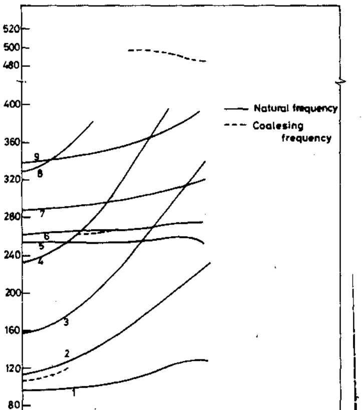

Fig. 2 Effect of rise on natural frequency for eight-layered cross-ply panels, $a/b = 2$, $b/h = 100$, $[0/90]_4$, $E_{11}/E_{22} = 25$, $G_{12}/E_{22} = 0.5$, $\nu_{12} = 0.25$.

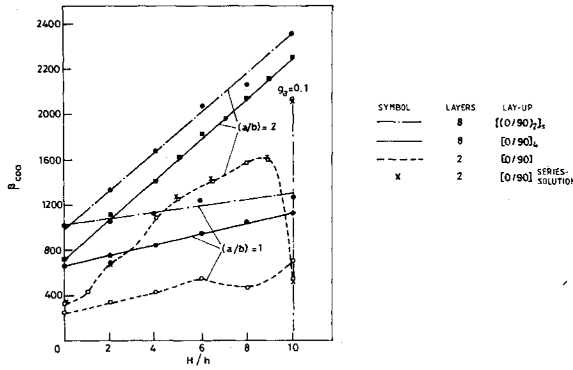

Fig. 3 Rise vs critical flow parameter for multilayered cross-ply panels, $b/h = 100$, $E_{11}/E_{22} = 25$, $G_{12}/E_{22} = 0.5$, $\nu_{12} = 0.25$.

In Fig. 3, the variation of β_{coa} vs (H/h) is plotted for the panels [with $a/b = 2$ and 1; number of layers being 2 (asymmetric) and 8 (symmetric and asymmetric)]. In the case of eight-layered panels, the variation of flutter bound β_{coa} is almost linear. In the case of two-layered panels, this regularity is disturbed. There is a sharp dip in the case of a two-layered panel with $a/b = 2$ and $H/h = 10$, thus indicating a possibility of snap buckling. However, when damping ($g_a = 0.1$) is taken into consideration (i.e., two-dimensional quasisteady aerodynamic approximation), the flow parameter is shifted to $\beta_{cr} = 2333$ from $\beta_{coa} = 515.6$, while the critical frequency parameter remains at 307.0. It was verified that series solution also gave the same result for this case. This phenomenon (i.e., the addition of a slight amount of damping raises the flutter boundary considerably) is similar to what has been observed by Ramkumar and Weishaar[8] in the case of anisotropic plates, and by Matsuzaki[7] in the case of isotropic circular cylindrical shell panels.

From Fig. 3 it can be observed that the flutter bound β_{coa} obtained using two-dimensional quasistatic aerodynamic theory, for any particular panel, is in the increasing order corresponding to the following cases of lamination, i.e., two-layered asymmetric, eight-layered asymmetric, eight-layered symmetric. The bending/stretching coupling due to material lamination will be in the descending order for the above cases, i.e., highest for two-layered asymmetric, zero for eight-layered symmetric. Hence, it can be concluded that the greater the coupling due to lamination, the lower is the value of β_{coa}.

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